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ROBUST, ADAPTIVE, AND NETWORK CONTROL

A Comparative Analysis of Centrality Measures in Complex Networks

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Abstract—Identification of central elements in networks is an ill-defined problem. Hence, a large number of centrality measures have been proposed in the literature. We present a survey of existing axioms, which characterize certain properties of centralities. We also perform a perturbation analysis of centrality measures in real and artificial networks.

Keywords: network analysis, centrality, axioms for centrality, perturbation analysis

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1. INTRODUCTION

Network models are essential tools for modeling and studying complex systems. Networks can describe most existing technological, biological, social, and other systems, as well as model various socio-economic, transportation, epidemiological, industrial and other processes.

One of the fundamental challenges in network science is the detection of the most important participants [1]. Unfortunately, the identification of central elements is an ill-defined problem because there is no unique definition of importance. In general, the notion of importance depends on the problem statement and the nature of a network. Recently, a large number of centrality models have been proposed in the literature based on the number of connections of each node, the number of paths between them, individual attributes of the nodes and their influence in the group, etc. [2]. However, since the results of the centrality measures vary significantly, it is important to compare these models and estimate the quality of the obtained results.

Comparison of centrality indices is a complex task due to the subjective nature of importance in networks. In several studies, centrality measures can be compared with ground truth or expert knowledge about the real influence of nodes [3, 4]. In that case, the centrality measure, which best approximates the predefined influence, will be selected for further use in that network (e.g.: to identify central nodes in the same network at the next discrete time or to define the centrality of unlabeled nodes). Unfortunately, ground-truth data are extremely rare and difficult to collect in real-world networks.

Another way to compare centrality measures is to check the properties (called *the axioms of centrality*) that these measures satisfy [5–13]. The axioms provide insight into the performance of centrality indices and help to understand whether a particular set of nodes will be identified in a certain graph topology. Hence, the axioms of centrality assess how well a given centrality measure satisfies a set of logical properties. However, the current list of axioms is neither unified nor exhaustive, leading different studies to focus on various sets of properties.

An important aspect of centrality models is their robustness to small perturbations in a network structure. Since many real-world networks are incomplete or contain incorrect data (errors/missing data), the results of many centrality measures may be biased. There are several studies in the literature that investigate the sensitivity of various centrality models [14–19]. Nevertheless, most of these studies are limited to the analysis of only classical centralities on a small set of networks.

This study aims to provide an overview of the existing axioms that characterize the concept of centrality in network structures. We also analyze the sensitivity of centrality measures to small changes in a network. The results of this research provide a deeper understanding of how incorrect and incomplete data affect the results of existing centrality measures.

The paper is organized as follows. Section 2 describes the centrality models that we study in the paper. Section 3 provides an overview of the existing axioms for centrality measures. In Section 4 we evaluate the robustness of centrality measures to small perturbations in classical and real-world network structures. The final section concludes.

2. CENTRALITY MODELS IN NETWORK STRUCTURES

Consider a graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ is a set of nodes, $|V| = n$, and $E \subseteq V \times V$ is a set of links between nodes. The graph G is *undirected* if $\forall i, j \in V\colon (i, j) \in E \Longrightarrow (j, i) \in E$ and *directed* otherwise. The graph G can be represented by $n \times n$ adjacency matrix $A = [a_{ij}]$, where $a_{ij} = 1$ if $(i, j) \in E$, otherwise $a_{ij} = 0$. The graph G is *weighted* if it can be described by a non-negative weight matrix $W = [w_{ij}]$, where w_{ij} represents the intensity of a link between nodes i and j. Additionally, every node may be associated with a set of attributes $\{w_i^k\}$, where $i \in V$ and k is an attribute number, $k \in K$, and threshold value q_i, which denotes the level when node i becomes affected by other nodes. *Centrality score* of a node i is a certain numerical value $c_i \in \mathbb{R}^+$ that characterizes the i's importance in graph G.

Centrality is one of the key concepts in complex networks. There exist many measures to identify the most central elements in a network [1–3]. We provide a description of the centrality measures that we study in the paper.

2.1. Degree Centralities

The simplest centrality measure of node i in undirected graph is the degree [20], which is the number of links that are incident to node i:

$$c_i = \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji}. \tag{1}$$

For directed graphs, there exist two variants of degree: in-degree and out-degree. The in-degree of a node is the number of in-coming links to this node, while the out-degree is the number of out-coming links from this node. Degree centrality can also be adapted to weighted networks (w_{ij} substitutes a_{ij}).

2.2. Spectral Centralities

Degree centralities are local measures that do not take into account indirect connections between nodes. The generalization of a degree centrality is the eigenvector centrality [20] that considers both direct and indirect connections between nodes. The eigenvector centrality $\vec{c}$ assigns the relative importance of node i as follows: more important neighbors should contribute more to the centrality of node i than less important neighbors. The calculation of the eigenvector centrality reduces to the eigenvector problem of the adjacency matrix A:

$$A\vec{c} = \lambda\vec{c}, \tag{2}$$

where λ is the principal eigenvalue of matrix A. In practice, the eigenvector centrality is used for undirected graphs because the estimation of the eigenvector for directed graphs may result in complex eigenvalues.

Another example of spectral centrality measure is PageRank [21], which evaluates the probability of visiting each node by the random walker:

$$c_i = \alpha \sum_{j=1}^{n} \frac{c_j}{\sum_{k=1}^{n} a_{jk}} a_{ji} + \frac{1-\alpha}{n}, \tag{3}$$

where α is a teleportation coefficient, $0 \leqslant \alpha \leqslant 1$. PageRank can be applied to undirected and directed graphs. Other centrality measures based on the eigenvector calculation also include Katz centrality [22], the HITS algorithm (hubs and authorities) [23], the subgraph centrality [24].

2.3. Centralities That Are Based on the Shortest Paths

Another class of centrality measures is based on the shortest paths between nodes. The most common centrality is a closeness centrality [20, 25] that estimates how close node i is located to other nodes in a graph:

$$c_i = \frac{1}{\sum_{j=1}^{n} d_{ij}}, \tag{4}$$

where d_{ij} – the length of the shortest path from node i to node j.

The betweenness centrality also considers the shortest paths between nodes [26, 27]. This measure reflects the importance of a node i as an intermediary between all pairs of other nodes:

$$c_i = \sum_{j \neq k} \frac{\sigma_{jk}(i)}{\sigma_{jk}}, \tag{5}$$

where σ_{jk} is the number of the shortest paths from node j to node k and $\sigma_{jk}(i)$ is the number of the shortest paths from j to k that include node i.

2.4. Short- and Long-Range Interaction Centralities

Many centrality measures do not consider individual attributes of nodes and/or the possibility of their group influence. Moreover, many existing methods do not assess the intensity of distant connections or take into account insignificant connections between nodes. The Long-Range Interaction Centrality (LRIC) is proposed in [3, 28] to account for these features.

Denote by N_j the set of neighbors of node j. The group of nodes $\Omega(j) \subseteq N_j$ is called *critical* for node j if $\sum_{i \in \Omega(j)} w_{ij} \geqslant q_j$. Node $l \in \Omega(j)$ is called *pivotal* if its exclusion from the critical group $\Omega(j)$ makes this group non-critical. All pivotal nodes in group $\Omega(j)$ is denoted by $\Omega_p(j)$. Then the influence of node i on node j is defined based on the critical group $i \in \Omega_p(j)$, in which node i has the maximum influence, i.e.

$$c_{ij} = \begin{cases} \max\limits_{\Omega_p(j): i \in \Omega_p(j)} \dfrac{w_{ij}}{\sum\limits_{k \in \Omega_p(j)} w_{kj}}, & \text{if } \exists \Omega_p(j)\colon i \in \Omega_p(j), \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Value c_{ij} is the element of an $n \times n$ matrix $C = [c_{ij}]$, which can be further used to determine indirect influence between nodes. The LRIC index has several modifications. In particular, there exists the Short-Range Interaction Centrality (SRIC) index [29] as well as the bundle index [30]. LRIC demonstrated its effectiveness for many real networks including foreign claims, international migration, world trade, global food network, the citation network of economic journals and the networks of interactions between terrorist groups [3, 31, 32].

3. PROPERTIES OF CENTRALITY INDICES: LITERATURE REVIEW

Several studies compare centrality measures with respect some axioms, that determine a set of logical properties that a given centrality measure should satisfy [5–11]. Section 3 discusses the axioms of centrality and provides a list of centrality indices for which these axioms have been studied.

3.1. Anonymity Axiom

The anonymity axiom is proposed in [5]. Anonymity axiom is satisfied if the centrality of node i only depends on its position in a network, i.e.

$$c_i(V,E) = c_{f(i)}(V, \{(f(i), f(j))\}\colon (i,j) \in E), \tag{7}$$

where $c_i(V,E)$ is the centrality score of node i in graph $G = (V,E)$ and f is a bijection on a set of nodes $f{:}V \to V$. In other words, the centrality of a node remains the same as the centrality of the corresponding node in the isomorphic graph. We note that all centrality indices that do not use nodes attributes satisfy the anonymity axiom.

3.2. Endpoint Increase Axiom

The endpoint increase axiom [5] is satisfied if the addition of a link between two nodes does not decrease their centralities, i.e. $\forall G,\ \forall i,j \in V$

$$\begin{cases} c_i(V, E \cup (i,j)) \geqslant c_i(V,E), \\ c_j(V, E \cup (i,j)) \geqslant c_j(V,E), \end{cases} \tag{8}$$

where $c_i(V, E \cup (i,j))$ is a centrality value of node i in graph G' which is formed by adding a link (i,j) to the graph $G = (V,E)$. Thus, any pair of non-adjacent nodes has an incentive to establish a connection between them.

3.3. Monotonicity Axiom

The monotonicity axiom [5] is satisfied if the addition of link (i,j) does not decrease the centrality of any node, i.e. $\forall G,\ \forall i,j,k \in V$

$$c_k(V, E \cup (i,j)) \geqslant c_k(V,E). \tag{9}$$

If a centrality measure involves the normalization at the last step, then the monotonicity axiom will most likely not be satisfied, as an increase in the centrality of one node leads to the decrease in the relative centrality of other nodes. Furthermore, if the endpoint increase axiom is not satisfied for a given centrality measure, then the monotonicity axiom also does not hold.

3.4. Top Node Axiom

According to the top node axiom [5], if a node i has the highest centrality in the initial graph G then adding an incident link will not change its position, i.e.

$$c_i(V,E) \geqslant c_k(V,E)\ \forall k \in V \implies \forall j,k \in V\ c_i(V, E \cup (i,j)) \geqslant c_k(V, E \cup (i,j)). \tag{10}$$

Thus, the top node axiom is not satisfied if the addition of incident links decreases the centrality of the most central node, which contradicts the intuitive concept of a centrality in a network.

3.5. Fairness Axiom

The fairness axiom [6] is satisfied if the addition of a link (i, j) changes the centrality of incident nodes by the same value, i.e. $\forall G,\ \forall i, j \in V$

$$c_i(V, E \cup (i, j)) - c_i(V, E) = c_j(V, E \cup (i, j)) - c_j(V, E). \tag{11}$$

Thus, the contribution of a new connection should be the same for each of the incident nodes.

3.6. Balanced Contributions Axiom

The balance of contributions axiom is proposed in [7] and is axiomatized in [9]. A centrality measure satisfies the axiom if removing all links E_i that are incident to node i affects the centrality of node j in the same way as removing all links E_j affects the centrality of node i, i.e. $\forall i, j \in V$

$$c_i(V, E) - c_i(V, E \backslash E_j) = c_j(V, E) - c_j(V, E \backslash E_i), \tag{12}$$

where E_i is a set of links that are incident to node i.

3.7. Add Edge Distance Axiom

The edge distance axiom is proposed in [9]. According to this axiom, the addition of a new link between nodes i and j, which are located at the same distance from node k, does not affect the centrality of node k, i.e. $\forall i, j, k \in V\colon d_{ik} = d_{jk}$

$$c_k(V, E) = c_k(V, E \cup (i, j)). \tag{13}$$

Thus, the addition of a link between nodes should not affect the centrality of other nodes that are at the same distance from these nodes.

3.8. Bridge Axiom

According to the bridge axiom [9], if a link (i, j) is the only connection between two disconnected components of the graph (i.e., (i, j) is a bridge), then the centrality of a node i should be higher than the centrality of a node j if the component that contains node i is larger than the component with node j.

3.9. Cut-Vertex Additivity Axiom

According to the cut-vertex additivity axiom [10], if two graphs G and G', which share only one common node i, are merged, then the centrality of node i equals the sum of its centrality in graphs G and G', i.e. $\forall G = (V, E),\ G' = (V', E')\colon V \cap V' = \{i\}$

$$c_i(V \cup V', E \cup E') = C_i(V, E) + C_i(V', E'). \tag{14}$$

Alternatively, this centrality can be formulated as follows:

$$\frac{1}{c_i(V \cup V', E \cup E')} = \frac{1}{c_i(V, E)} + \frac{1}{c_i(V', E')}. \tag{15}$$

The next two axioms are formulated in the context of specific graph structures.

3.10. Density Axiom

Consider the graph $G_{k,p}$, which consists of a complete graph with k nodes connected by a single link (i, j) to a cycle of length p ($p \geqslant 3$). Suppose that node i is from the complete graph and node j is from the cycle. The density axiom [8] is satisfied if for $k = p$ the centrality of node i is greater than the centrality of node j.

3.11. Size Axiom

Consider the graph $G_{k,p}$, which consists of a complete graph with k nodes connected by a single edge (i, j) to a cycle of length p ($p \geqslant 3$). The size axiom [8] is satisfied if for any k there exists a value $p < k$ such that the centrality of the nodes in the cycle is higher than the centrality of the nodes in the complete graph, and conversely, for any k there exists a value $p \geqslant k$ such that the centrality of the nodes in the cycle is lower than the centrality of the nodes in the complete graph.

Many axioms have already been studied for certain centrality models (see Table 1) [33]. More details about the properties of centrality measures from Section 2 are discussed in [8, 10, 11, 34]. We remark that even classical centrality measures have been studied only partially.

Table 1 shows that only degree centrality satisfies most axioms. This observation is due to the fact that degree centrality is a local measure (it considers only direct connections between nodes). Nevertheless, most of the axioms focus on the global structure of a network and assess the impact of indirect connections between nodes. Since changes in links do not affect the degree of distant nodes, degree centrality satisfies most axiomatic properties. Other centrality models do not satisfy most axioms. All measures, which do not consider individual attributes of nodes, satisfy the anonymity axiom. The endpoint increase and monotonicity axioms are only satisfied by closeness and Katz centralities while the density axiom is satisfied by eigenvector, Katz, PageRank and closeness centralities.

The presented list of axioms is not exhaustive. Other axioms can be also found in [35–38]. We note that many axioms from other related scientific fields could be adjusted to centrality metrics. For instance, consider the *Concordance axiom* for the choice functions [39]. According to this axiom, an alternative chosen in two subsets should also be chosen on their union. In choice theory, this condition characterizes the rational choice of the set of alternatives. Consequently, we can

Table 1. Properties of centrality measures ("+" – the axiom is satisfied, "–" – the axiom is not satisfied, "?" – the axiom have not been studied in the literature yet)

No.	Models of centrality	Anonymity	Endpoint Increase	Monotonicity	Top node	Fairness	Balanced Contribution	Add Edge Distance	Bridge	Cut-vertex	Density	Size
1	Degree centralities	+	+	+	+	+	+	+	–	+	+	–
2	Eigenvector	+	–	–	–	–	–	–	–	–	+	–
3	PageRank	+	?	–	?	–	–	–	–	–	+	–
4	Katz centrality	+	+	+	?	–	–	–	–	–	+	–
5	HITS	+	?	?	?	–	–	?	?	?	?	?
6	Subgraph centrality	+	?	?	?	–	–	?	?	?	?	?
7	Closeness centrality	+	+	+	–	–	–	+	+	–	+	–
8	Betweenness centrality	+	–	–	–	–	–	–	–	–	–	–
9	LRIC	–	?	–	?	–	–	?	?	?	?	?

extend the cut-vertex additivity axiom by assuming that if two graphs G and G' share a common node i, which has the highest centrality score in G and G', then node i should also be the most central node in their union.

We would like to emphasize that axioms of centrality helps to understand the properties and the behaviour of the centrality measure. However, if a specific centrality model does not satisfy certain axioms, we cannot conclude that this centrality measure is defective.

4. ROBUSTNESS OF CENTRALITY MEASURES

The robustness of centrality measures is one of the most important criteria for selecting an appropriate centrality model in the context of real-world systems. Unfortunately, many existing networks are incomplete or contain errors in data. For instance, criminal networks, which are discussed in [40, 41], are incomplete (due to the nature of the network), contain errors (unintentional data collection errors and intentional deception by criminals) and inconsistent information (misleading information from different sources). The network of foreign bank claims, which is analyzed in [31], covers about 94% of total foreign claims, as some countries do not report their statistics. Meshcheryakova [42] investigated the global food trade network under asymmetry that occurs due to different commodity classification systems, trade volume assessment and time lag. The results of centrality measures in these networks require a careful examination because many measures may be sensitive to small changes in the graph structure.

4.1. Robustness of Centrality Measures in Classical Graph Structures

We evaluate the sensitivity of centrality measures to link removal or addition to undirected unweighted networks. We perform the experiments on classical graph structures with $n = 100$ nodes (see Fig. 1):

1. Erdos-Renyi graph (ER graph): a random graph model $G(n, p)$ where n in a number of nodes and p is a probability for edge creation.

2. Barabasi-Albert graph (BA graph): a random graph model where nodes are sequentially connected to m existing nodes with a probability that is proportional to the degree of the existing nodes.

3. Small-world (Watts-Strogatz) graph model: a random graph model where a regular ring graph n vertices is created and then some links are randomly rewired with probability p.

4. Square lattice: a graph with n nodes whose nodes are the points in the Cartesian plane. Nodes are connected if the distance between them is 1.

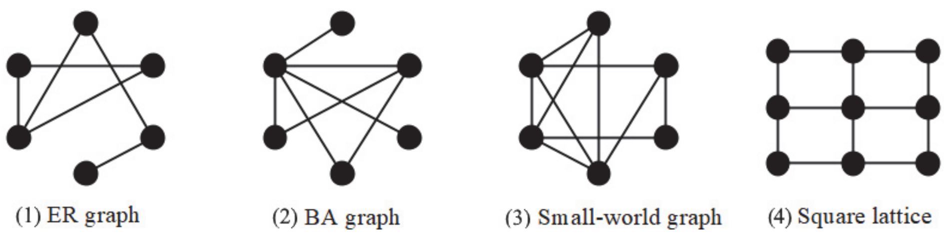

Fig. 1. Classical graph structures.

The computational experiments are performed on classical graph structures. First, $N = 100$ graphs of a given structure are generated ($N = 1$ graph for a square lattice). We consider the parameter $p = 0.05$ for random graph models and $m = 4$ for BA graphs[1]. For each graph, we

[1] The authors also considered other values of p (from 0.01 to 0.1 with a step of 0.01) and m (from 2 to 5). We observe that the relative position of the centrality measures in terms of their robustness does not change significantly for other parameter values.

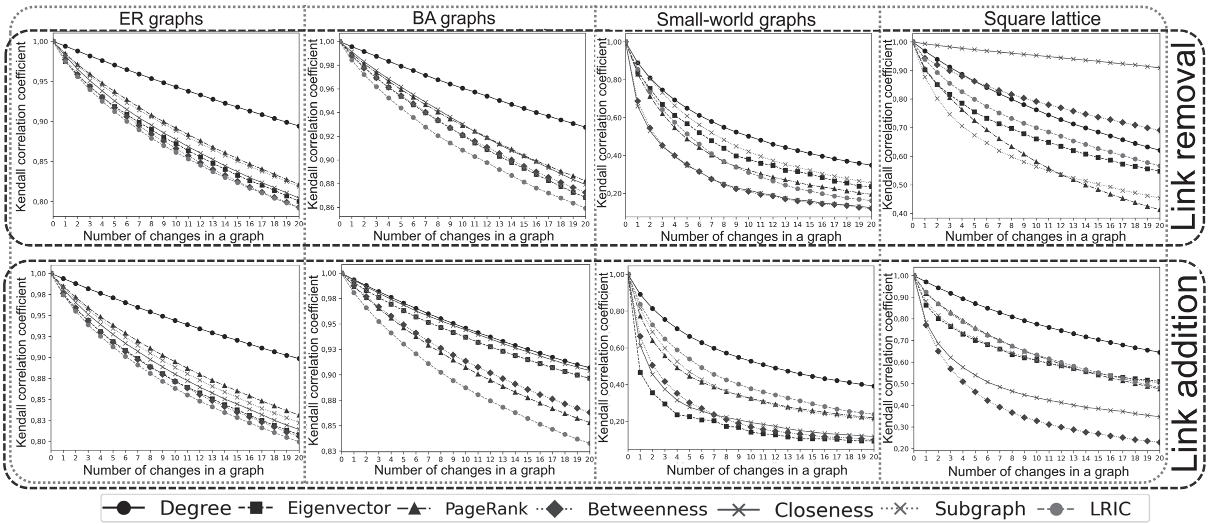

Fig. 2. Stability of centrality measures on classical graph structures.

examine $K = 100$ ways ($K = 10\,000$ for a square lattice) to change the graph structure. Next, we assess the centrality of the nodes in original and modified graphs and compare the centrality scores with respect to the Kendall correlation coefficient. The sensitivity of the robustness of a centrality measure is defined as the arithmetic mean across all 10 000 experiments. We remark that the increase in the number of experiments does not change the results significantly.

The computational experiments are carried out for the following centrality models: degree centrality, eigenvector centrality, PageRank, closeness centrality, betweenness centrality and LRIC. The results are provided in Fig. 2.

Figure 2 shows that the centrality measures are very robust in ER and BA graphs: the Kendall correlation coefficient remains high (> 0.8) after deleting/adding 20 links. However, small perturbations in small-world graphs and a square lattice dramatically change the centrality scores of the nodes. Thus, the centrality measures on these networks require a careful examination as most of them are very unstable. The exceptions are the degree centrality (link addition) and the closeness centrality (link removal in the square lattice graph).

Among the centrality measures, which consider the global structure of the network and have a low sensitivity to graph perturbations, we remark PageRank and subgraph centrality (ER, BA and small-world graphs) as well as LRIC (square lattice). The eigenvector and centralities, which are based on the shortest paths, are the most susceptible to random changes in most network structures. In general, the choice of the most appropriate centrality model depends on the network structure under consideration.

4.2. Robustness of Centrality Measures in Real Networks

In many applications, the network data is complete. Consequently, the need to assess the reliability and representativeness of the results obtained occurs in real networks. This section discusses the sensitivity of centrality models in criminal and international trade networks, where the structure is only partially observed.

Criminal network

The criminal network describes Sicilian Mafia meeting interconnections, which is derived from Court reports in 2007 based on the results of the anti-mafia “Montagna Operation.” The mafia meetings network is an undirected graph with 101 nodes and 256 links [41]. We assume that

Table 2. Robustness of centrality measures in criminal network

Models of centrality	Scenario 1		Scenario 2	
	Correlation	TOP-5	Correlation	TOP-5
Degree centrality	0.91	0.98	0.92	0.98
Eigenvector centrality	0.89	0.85	0.89	0.84
PageRank	0.78	0.94	0.85	0.93
Katz centrality	0.91	0.95	0.9	0.94
Subgraph centrality	0.92	0.91	0.92	0.88
Closeness centrality	0.81	0.88	0.83	0.87
Betweenness centrality	0.66	0.84	0.71	0.85
LRIC	0.74	0.8	0.79	0.81

Table 3. Robustness of centrality measures in global food trade network

Models of centrality	Scenrio 1		Scenrio 2	
	Correlation	TOP-10	Correlation	TOP-10
In-degree centrality (total import)	0.99	0.99	0.38	0.22
Out-degree centrality (total export)	0.99	1	0.79	0.92
Degree difference (net export)	0.99	1	0.14	0.81
Hubs	0.99	1	0.78	0.72
Authorities	0.99	1	0.26	0.22
PageRank	0.99	1	0.43	0.26
LRIC	0.99	0.98	0.71	0.5

approximately 10% of the total number of recorded connections are missing in the network and consider two strategies for adding new links: random addition (scenario 1) and the addition of new links with a probability that is proportional to the distance between nodes (scenario 2). The sensitivity of centrality measures is assessed using the average Kendall correlation coefficient between node rankings and the average Jaccard coefficient between the sets of TOP-5 nodes in original and modified networks. The results are depicted in Table 2.

Table 2 demonstrates that random changes in the graph structure (scenario 1) have a greater impact on the ranking of nodes compared to changes based on the distance between nodes (scenario 2). The degree centrality shows the most robust results, which is consistent with the findings in Section 4.1. Katz centrality and subgraph centrality, which are global measures, also provide a robust ranking of nodes. In contrast, the results of LRIC and shortest path-based centralities are very sensitive to missing links in a criminal network.

Global food trade network

The global food trade network represents trade flows between 222 countries in 2020 [43]. Since the network is directed and weighted (the largest trade flow is between the USA and Canada – about 5.5 billion dollars), we focus on centrality measures that are usually applied to trade networks. We consider the following strategies to change the graph structure:

(1) Scenario 1: modifying the recorded trade flows by up to 5% to account for possible errors in estimating trade flows;

(2) Scenario 2: adding links with a total value of 1% of the total world trade to account for missing flows due to the lack of reporting by some countries.

The sensitivity of centrality measures is assessed using the average Kendall correlation coefficient between node rankings and the average Jaccard coefficient between the sets of TOP-10 countries in original and modified networks. The results of the performed analysis are provided in Table 3.

Table 3 shows that errors in trade flows within 5% (scenario 1) have virtually no effect on the ranking of countries by all centrality indices. However, the lack of trade flow data between some countries (scenario 2) can significantly affect the ranking of countries. In particular, the highest correlation coefficient (> 0.71) is observed for out-degree centrality, LRIC, and hubs (part of the HITS algorithm). The low correlation coefficient can be explained by significant changes in the ranking of countries that were not the most important in the original network. Nevertheless, the set of the most influential countries (TOP-10) according to out-degree centrality, degree difference and hubs remains quite stable.

5. DISCUSSION

The concept of centrality in networks lacks a unique definition, leading to the development of many different centrality models. In this regard, selecting the most appropriate centrality measure often involves using an axiomatic approach and analyzing the sensitivity of models to changes in network structure.

Our study provided an overview of the existing axioms of centrality and current results on verifying these axioms for classical centrality measures. Many properties of centrality measures remain unexplored. Hence, we urge other researchers to examine the axiomatic properties of these measures. Studying the axioms could also be valuable for the development of new centrality models.

We also compared the sensitivity of centrality measures to small perturbations in a network structure. As most existing networks change over time and their structure is only partially observed, the question arises about the representativeness and reliability of the resulting set of central elements. We demonstrated the robustness of classical centrality models to changes in the structure of both artificial and real networks. The results of the models that are highly sensitive to changes in network structure require a careful examination, especially for partially observed networks.

Our study expands the theoretical knowledge of the concept of centrality in network structures. Our research findings can help choose the most suitable centrality index, especially in cases where the observed network is incomplete or inaccurate. Finally, the results of the study can be utilized to develop new centrality measures.

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